'PROJECTILE MOTION'

If velocity of particle in one direction is const & Acceleration which is in $\perp$ direction Remain same w.r.t Motion of particle is called projectile & It path is always parabolic.

Ground – Ground projection

$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g} \leftarrow \text{Time of flight.}$$

$$h_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g} \rightarrow \text{max Height.}$$

$$R = (u\cos\theta)T = \frac{2u_x u_y}{g} = \frac{u^2\sin\theta}{g}$$

$$\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$\alpha = (u\cos\theta t)\hat{i} + (u\sin\theta t - \tfrac{1}{2}gt^2)\hat{j}$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left[1 - \frac{x}{R}\right]$$

* From bottom to top $\alpha \downarrow$
* At top position $\alpha = 0$
* From top to bottom $\alpha \uparrow$

* Horizontal Range. $\boxed{R = \frac{u^2\sin 2\theta}{g}}$

* K.E at lowest point. $\boxed{KE = \frac{1}{2}mu^2}$

* velocity at time 't'

Horizontal

$\boxed{V_x = u\cos\theta \propto t^0}$

Vertical

$\boxed{V_y = (u\sin\theta) - gt}$

* Angle of velo. from x-axis

$$\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$$

* Disp. at time 't'

$$|\vec{r}| = \sqrt{x^2 + y^2}$$

NOTE → * Horizontal component of velocity Remain same all points of Its path

* At top point path of particle is circular & Resultant velo. Remain same.

Special case for Ground to Ground projection

Case-I → Max Range condition

$$R = \frac{u^2\sin 2\theta}{g}$$

* $u = c,\ g = c \Rightarrow \underset{\text{max}}{R} = f(\theta)$

$(\sin 2\theta) = 1$

$\Downarrow$

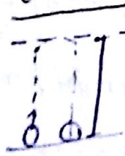
$2\theta = 90^\circ, \pi/2$

$\boxed{\theta = 45^\circ/\pi/4}$

$\boxed{R_{max} = \frac{u^2}{g} = R_{45^\circ}}$

$\boxed{h_{45^\circ} = \frac{u^2\sin^2 45^\circ}{2g} = \frac{u^2}{4g}}$

$\theta = 90^\circ$ (vertical projection)



$\boxed{R_{90} = 0}$ $\quad h_{90^\circ} = \frac{u^2}{2g} = \frac{R_{45^\circ}}{2} \Rightarrow \boxed{R_{45^\circ} = 2h_{90^\circ}}$

$\boxed{T_{max} = \frac{2u}{g} = T_{90^\circ}}$ $\quad \boxed{h_{45^\circ} = \frac{R_{45}}{4} = \frac{h_{90}}{2}}$

$R_{max} \Rightarrow \theta \Rightarrow 45^\circ$

$\left.\begin{matrix} H_{max} \\ T_{max} \end{matrix}\right] \Rightarrow \theta = 90^\circ$

(a) → $0 \le \theta \le 45$

$\theta\uparrow, T\uparrow, h_{max}\uparrow, R\uparrow$

(b) → $45^\circ < \theta \le 90^\circ$

$\theta\uparrow, T\uparrow, h_{max}\uparrow, R\downarrow$

Case-II Same Range condition

* $R_\theta = R_{90°-\theta} = \boxed{\frac{u^2 \sin 2\theta}{g}}$
* $R_{45°+\theta} = R_{45°-\theta} = \boxed{\frac{u^2 \sin 2\theta}{g}}$
* $T_\theta \neq T_{90-\theta}$, $T_{90} = \boxed{\frac{2u\cos\theta}{g}}$ $\boxed{T_\theta = T_{90°} = \tan\theta : 1}$
* $h_\theta \neq h_{90-\theta}$

$$\boxed{h_\theta : h_{90-\theta} = \tan^2\theta : 1}$$

$$\boxed{R = 4\sqrt{h_\theta h_{90-\theta}}}$$

* $R_{10°} = R_{80°} \Rightarrow h_{10°} < h_{80°} \Rightarrow T_{10°} < T_{80°}$
* $R_{30°} = R_{60°} \Rightarrow h_{30°} < h_{60°} \Rightarrow T_{30°} < T_{60°}$
* $R_{35°} = R_{55°} \Rightarrow h_{35°} < h_{55°} \Rightarrow T_{35°} < T_{55°}$

* $R_{10} < R_{30} < R_{30} < \overset{max}{R_{45°}}$
  $R_{10} = R_{80°}$, $R_{30} = R_{60°}$, $R_{30} = R_{55}$
* $h_{10°} < h_{30°} < h_{35} < h_{45} < h_{45°} < h_{60°} < h_{80°} < \underset{max}{h_{90°}}$

Case-III → Linear Momentum & change in Linear momentum.

$\vec{p}_i = m\vec{u}_i = m(u\cos\theta\,\hat{i} + u\cos\theta\,\hat{j})$

$p_f = m\vec{v}_f = m[(u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}]$

*change in momentum.

$$\boxed{\Delta p = mgt(\hat{j})}$$

NOTE → In Ground to Ground projection linear momentum remain same in horizontal direction. It change only in vertical direction.

(a) → B/w point of projection & top point.

$$\boxed{\Delta\vec{p} = mu\sin\theta(\hat{-j})} \quad \boxed{\Delta p_{horizontal} = 0}$$

(b) → B/W Ground to Ground point

$$\boxed{\Delta\vec{p} = 2mu\sin\theta(-\hat{j})}$$

Case-IV → Angular Momentum (J)

$$\boxed{\vec{J} = \vec{r} \times \vec{p}} \quad \boxed{J = \frac{mg(u\cos\theta)t^2}{2}(-\hat{k})}$$

NOTE → * Direction of Angular Momentum is always ⊥ to plane of Ground to Ground projection.

(a) → At top point of projectile w.r.t point of projection

$$\boxed{J = \frac{mu^3}{2g}\sin\theta\cos\theta(-\hat{k})}$$

(b) → At Ground point w.r.t point of projection.

$$\boxed{J = \frac{2mu^3}{g}\sin^2\theta\cos\theta(-\hat{k})}$$

case V → Energy of particle in protectile Motion

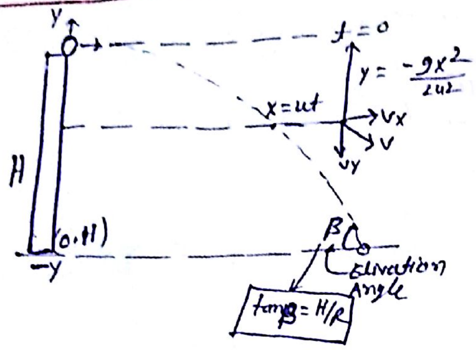
* $K \cdot E_{bottom} = \frac{1}{2} m u^2$
* $K \cdot E_{Top} = K \cdot E_B \cos^2\theta$
* $T \cdot M \cdot E = K \cdot E_B = \frac{1}{2} m u^2$
* $P \cdot E_t = \frac{1}{2} m u^2 - K \cdot E_t$
* $K \cdot E_t = \frac{1}{2} m \left[\sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}\right]^2$

NOTE →
* Average velo. of particle in ground to ground protection $u\cos\theta$

NOTE → In ⊕nce of gravitational force field total mechanical energy Remain same at all points of its path $(\frac{1}{2} m u^2)$

* In ground to ground protection vertical velocity at before last sec. of Ascending & After Ist second of descending Remain same & equal to $9.8\, m/sec.$ & vertical displacement is 5m (Independent from protection velocity and angle.)

Horizontal protection



|i| → velocity at time 't'
* Horizontal $\boxed{V_x = u}$ $K\, t^0$
* vertical $= -gt$

$$\boxed{|\vec{V}| = \sqrt{u^2 + 2gh}}$$

* Angle of inclination (x-axis)

$$\boxed{\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-gt}{u}\right)}$$

|ii| → position at time 't'

x-direction → $\boxed{X = ut}$

y → direction → $\boxed{y = -\frac{1}{2} g t^2}$

|iii| → Equation of path

$$\boxed{y = -\frac{gx^2}{2u^2}}$$ → parabola.

|iv| → Time of flight (T)

$$\boxed{T = \sqrt{\frac{2H}{g}}} \propto u^0$$

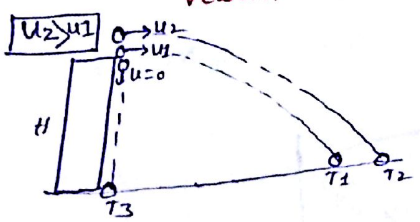
|v| → Range (R)

$$\boxed{X = ut}$$

$$\boxed{R = uT = u\sqrt{\frac{2H}{g}}}$$

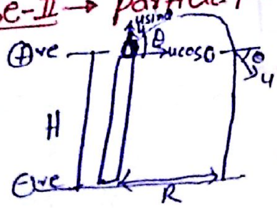
Special case

case-I → Time of flight in Horizontal protection Independent from protection velocity.



* $T_1 = T_2 = T_3 = \sqrt{\frac{2H}{g}}$
* $R_3 = 0$, $R_2 > R_1$
* Vt. velocity at bottom $V_{y1} = V_{y2} = V_{y3} = -\sqrt{2gh}$
* $V_b = \sqrt{u^2 + 2gh}$

case-II → Particle protected an angle 'θ' in upward direction from Horizontal.



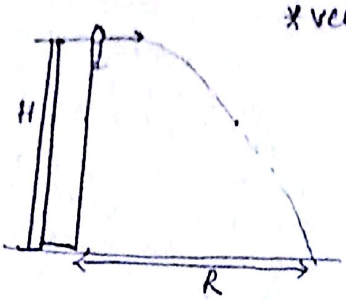
* Velocity at bottom $|\vec{V}| = \sqrt{u^2 + 2gh}$
* Time of flight $T = t_2 = \frac{u\sin\theta + \sqrt{u^2\sin^2\theta + 2gh}}{g}$ = ⊕ve

$$\boxed{T = \frac{|u_y| + |v_y|}{g}}$$

* Range of particle

$$\boxed{R = (u\cos\theta) T}$$

Case-III → Particle projected at an angle 'θ' from Horizontal in downward direction.



* velocity at bottom

$$\boxed{|V_b| = \sqrt{u^2+2gh}}$$

* Range

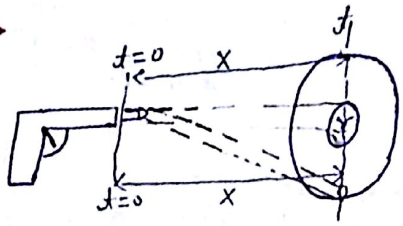
$$\boxed{R=(u\cos\theta)T}$$

* Time of flight

$$\boxed{T=\frac{-u_y+|V_y|}{g}}$$

Case →

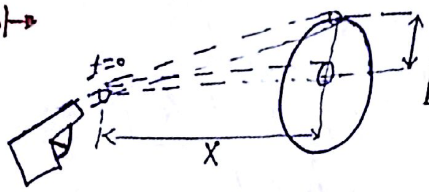
|a| →



*

$$\boxed{H = \tfrac{1}{2}gt^2 = \frac{gx^2}{2u^2}}$$

|b| →



$$\boxed{h=\frac{1}{2}gt^2=\frac{gx^2}{2u^2}}$$

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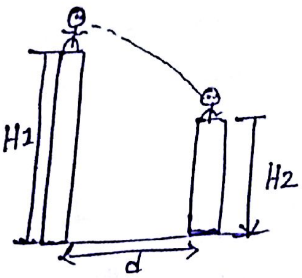

$$\boxed{\begin{aligned} R &= u\sqrt{\frac{2H}{g}} \\ u &= d\sqrt{\frac{g}{2(H_1-H_2)}} \end{aligned}}$$

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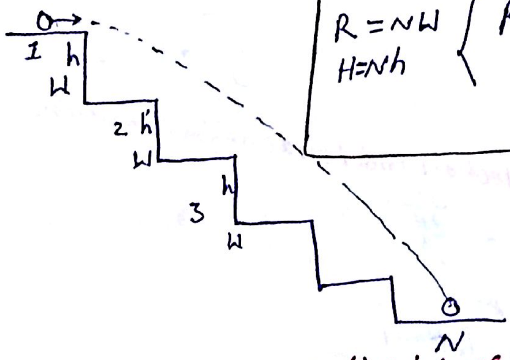


$$\boxed{\begin{aligned} R&=Nw \\ H&=Nh \end{aligned} \left\{ \begin{aligned} R&= u\sqrt{\frac{2H}{g}} \\ u&=w\sqrt{\frac{Ng}{2h}} \end{aligned} \right.}$$

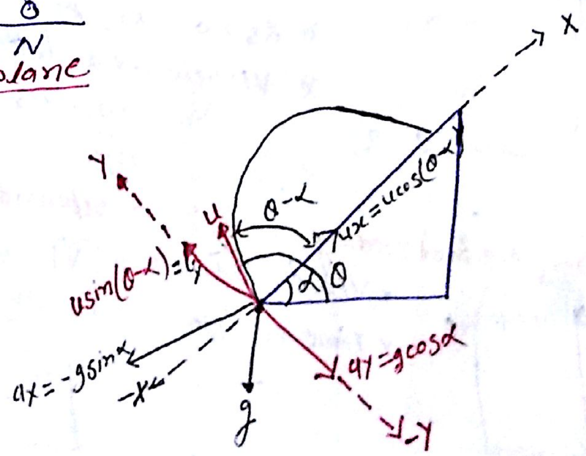
Projectile Motion on Incline plane

*Time of flight (bottom to top)

$$\boxed{t=\frac{2u\sin(\theta-\alpha)}{g\cos\alpha}=T}$$

* Range (R)

$$\boxed{R=\frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}}$$



(4)

Max Rangle condition

Upward (Bottom to Incline plane)

$$\boxed{\theta = \frac{\pi}{4} + \frac{\alpha}{2} \qquad R_{MAX} = \frac{u^2}{g(1-\sin\alpha)}}$$

Downward (Incline plane to bottom)

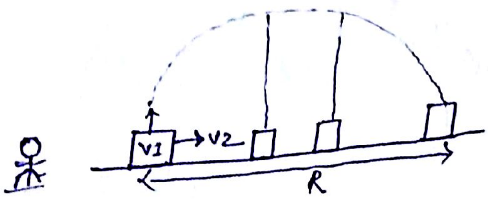
$$\boxed{\theta = \frac{\pi}{4} - \frac{\alpha}{2} \qquad R_{max} = \frac{u^2}{g(1+\sin\alpha)}}$$

Relative motion in Ground to Ground protection

$V = const$

$V_x = V_2$

$V_y = V_1$



$R = \frac{2U_yU_x}{g} = \frac{2V_1V_2}{g} = S_p = S_v$ ← distance of vehicle in Horizontal

↑ distance of particle in Horizontal.

$T = \frac{2V_2}{g} = \frac{2V_1}{g}$

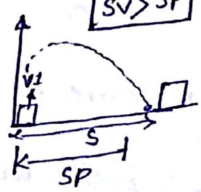
$H_{max} = \frac{V_2^2}{2g} = \frac{V_1^2}{2g}$

* When Accln = const.

$\boxed{R = \frac{2V_1V_2}{g}}$ $\boxed{T = \frac{2V_1}{g}}$

$\boxed{H_{max} = \frac{V_1^2}{2g}}$

* $\boxed{S_V > S_P}$



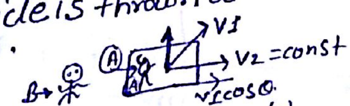
$V_2 \rightarrow$ velocity of vehicle at time of protetile

$V_1 \rightarrow$ velocity of particle.

* When Accln = -ve

$\boxed{S_V < S_P}$

* When particle is thrown at an angle 'θ' from vechile & velo. of vehicle is const.



	w.r.t 'A'	w.r.t 'B'
(i) → Initial Hz. velo.	$V_1\cos\theta + V_2$	$V_1\cos\theta$
(ii) → Initial vert. velo.	$V_1\sin\theta$	$V_1\sin\theta$
(iii) → T	$\frac{2V_1\sin\theta}{g}$	$\frac{2V_1\sin\theta}{2}$
(iv) → H	$\frac{V_1^2\sin^2\theta}{2g}$	$\frac{V_1^2\sin^2\theta}{2g}$
(v) → R	$(V_1\cos\theta + V_2)T$	$(V_1\cos\theta)T$

NOTE → When vehicle move c const. accn particle fall behind the observer & When vehicle move with const. Retardation than particle fall on Front of thrower.

Average velocity

1st method

$$\boxed{V_{avg} = \frac{\vec{v_1} + \vec{v_2}}{2}}$$

2nd method

$$\boxed{\vec{V}_{avg} = \frac{\vec{r}}{t} = \frac{x\hat{i} + y\hat{j}}{t}}$$

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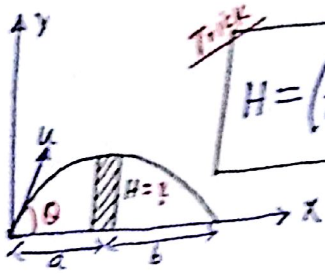
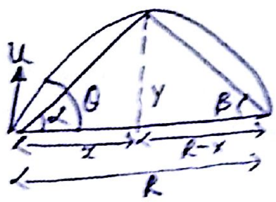


Trick

$$\boxed{H = \left(\frac{ab}{a+b}\right)\tan\theta}$$

Particle projected at an angle 'θ' from Horizontal as shown. base angle of triangle is α & β than value of $\tan\alpha + \tan\beta$.



*
$$\boxed{\tan\alpha + \tan\beta = \tan\theta}$$

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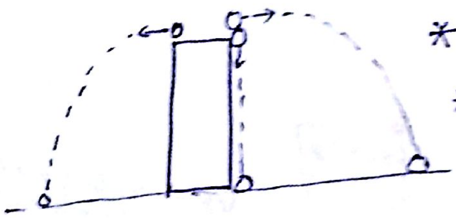

* $x = t_1 = t_2 = t_3 = \sqrt{\frac{2H}{g}}$

* $v_y = v_1 = v_2 = v_3 = -\sqrt{2gh}$

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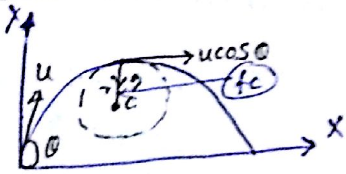
$$\boxed{t = \frac{dR}{V_R}}$$

→ Same direction 't' = $\boxed{\frac{L_1 + L_2}{|V_1 - V_2|}}$

→ Opposite direction 't' = $\boxed{\frac{L_1 + L_2}{|V_1 + V_2|}}$

Radius of curvature

|a| → At top point of projectile



$$r_{Top} = \frac{u^2\cos^2\theta}{g}$$

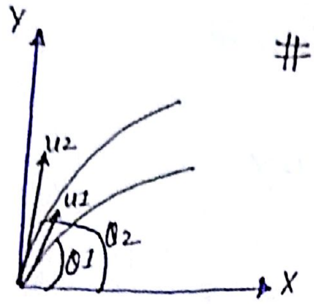
|b| → At bottom point of projectile.

$$\boxed{a_c = g\cos\theta = \frac{u^2}{r_b}}$$

$$\boxed{r_{bottom} = \frac{u^2}{g\cos\theta}}$$

$$\boxed{\begin{aligned} f_c = mg &= \frac{mv^2}{r_{Top}} \\ g &= \frac{(u\cos\theta)^2}{r_{Top}} \end{aligned}}$$

Path of projectile as seen from another projectile is straight line.



$x_R = u_R t + \frac{1}{2} a_R t^2$ ($a_R = 0$)

$$\boxed{x_R = (u_2\cos\theta_2 - u_1\cos\theta_1)t}$$

$y_R = u_R t + \frac{1}{2} a_R t^2$ ($a_R = 0$)

$$\boxed{\begin{aligned} y_R &= (u_2\sin\theta_2 - u_1\sin\theta_1)t \\ y_R &= \left(\frac{u_2\sin\theta_2 - u_1\sin\theta_1}{u_2\cos\theta_2 - u_1\cos\theta_1}\right)x_R \end{aligned}}$$

NOTE→ In Ground to Ground projection vertical disp. in last sec. of Ascending & 1st sec of descending is 5m & speed before last sec. of Ascending & After 1st sec. of descending is 10 m/sec.

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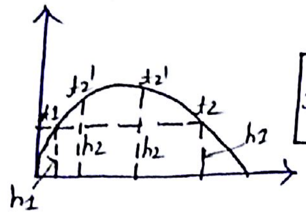

$$\boxed{t_1 + t_2 = t_1' + t_2' = T}$$

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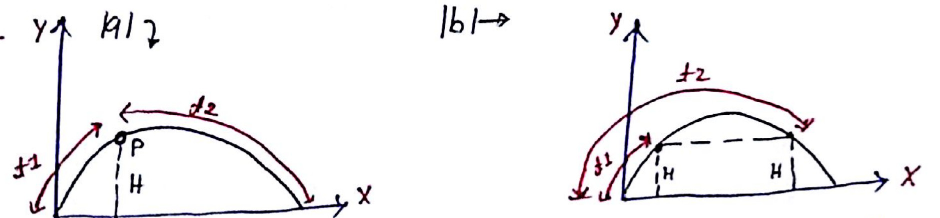


|a|→ $T = t_1 + t_2$

|b|→ $u_y = u\sin\theta = \frac{g}{2}(t_1 + t_2)$

|c|→ $H_{max} = \frac{g}{8}(t_1 + t_2)^2$

|d|→ $H = \frac{1}{2} g (t_1 t_2)$

Particle projected an angle 'θ' from Horizontal $\bar{c}$ speed 'u' cal. time taken by particle & Its velo when direction of velo. become ⊥ to its Initial velo. direction.

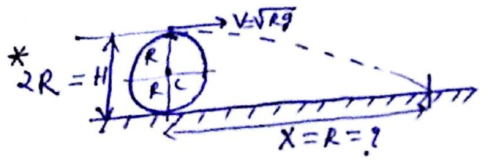
|1|→ $\boxed{t = \frac{u}{g\sin\theta} = \frac{u}{g}\operatorname{cosec}\theta}$

|2|→ $\boxed{v = u\cot\theta = \frac{u}{\tan\theta}}$

Particle / Water / Bullet are projected in different direction with same speed in horizontal plane. then max area covered by particle in Horizontal plane.

$$\boxed{A = \frac{\pi u^4}{g^2}}$$

Mud particle is attached with wheel of Radis 'R' is Rotate in vertical plane. After some time detach from wheel at top point of path with speed $\sqrt{Rg}$. then max Horizontal disp. of mud particle.



$X = \text{Range} = u\sqrt{\frac{2H}{g}}$

$= (\sqrt{Rg})\sqrt{\frac{2(2R)}{g}}$

$= \sqrt{4R^2}$

$\boxed{X = 2R}$

Two particle are projected Horizontal with speed u_1 & u_2 at same Instant from same height in opposite direction

(a) → Time taken by particle when its velocity become perpendicular $\boxed{t = \sqrt{\frac{u_1 u_2}{g}}}$

(b) → When velocity become perpendicular distance b/w particle. $\boxed{d = (u_1 + u_2)\sqrt{\frac{u_1 u_2}{g}}}$